Unit - I

Transportation models and Its variants

Defn of Transportation models:

The Transportation models is a special class of linear programming that deals with shipping of a commodity from sources (example, factory, stores, etc.,) to destinations (example, wharehouse). The objectives is to determine the shipping schedule that minimizes the total shipping cost while ~~stati~~ satisfying supply and demand.

Mathematically formulation:

Let there be m sources (origin) and n destination.

Let a_i be the number of units available in the ith sources.

Let b_j be the no. of units required in the j^{th} destination.

Let c_{ij} be the unit transporation cost from ith sources and j^{th} destination.

Let x_{ij} be the no. of units shipped from sources i to destination j.

minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$ subject to constraints

$\sum_{j=1}^{n} x_{ij} = a_i$, $i = 1, 2, \ldots m$, $\sum_{i=1}^{m} x_{ij} = b_j$, $j = 1, 2, \ldots, n$,

$x_{ij} \geq 0$ $\forall$ i and j.

Tabular representation:

A necessary and sufficient condition for feasible soln of a transportion problem is

$$\sum_{i=1}^{m} a_i = \sum_{i=1}^{n} b_j$$

Whare house / Factory	W_1	W_2	W_3	Avaibablity supply
F_1	c_{11} x_{11}	c_{12} x_{12}	c_{13} x_{13}	a_1
F_2	c_{21} x_{21}	c_{22} x_{22}	c_{23} x_{23}	a_2.
F_3	c_{31} x_{31}	c_{32} x_{32}	c_{33} x_{33}	a_3
Required Demand	b_1	b_2	b_3	Σa_i / Σb_i

A Set of non-negative decesion value x_{ij} Satisfies the constraint equation is called feasible solution.

A balanced transportion problem will always provided the feasible soln.

A feasible soln is said to be $\boxed{\text{basic}}$ if the no. of +ve allocations are m+n-1.

If the no. of allocation are less than m+n-1 is called degenerate basic feasible soln.

A feasible soln is said to be $\boxed{\text{optimal}}$ if it minimizes the total transportation cost.

The methods for finding the optimal problem, we consists of two steps:

(i) Find the initial basic feasible soln.

(ii) Find the optimal soln by making successive improvements from the initial basic feasible soln.

To find the initial basic feasible soln:

1. North-west corner method.
2. least cost method (or) matrix minima method.
3. vogel approximation method.

North-west Corner method:

The method starts at the north-west corner cell of the table (variable x_{11}).

Step : 1

Allocate as such as possible to the selected cell and adjust the associated amounts of supply and demand by subtracting the allocated amount.

Step : 2

Cross out the row or column with zero supply or demand to indicate that no further assignments can be made in that row or column. If both the row and column not to zero simultaneously, cross out one only, and leave a zero supply (demand) in the uncrossed-out row (column)

Step : 3

It exactly one row or column is left uncrossed out, stop. otherwise, move to the cell to the right if a column has ~~just~~ just been crosed or the one below if a row has been crossed out.

Go to step : 1

Ex:

The application of the procedure to the model.

~~gives~~	1	2	3	4	Supply
1	$\boxed{5}$ 10	$\boxed{10}$ 2	20	11	~~15~~ 10
2	12	$\boxed{5}$ 7	$\boxed{15}$ 9	$\boxed{5}$ 20	~~25~~ ~~20~~ 5
3	4	14	16	$\boxed{10}$ 18	10
Demand	~~5~~	~~15~~ 5	~~15~~	~~15~~ 10	50

$\boxed{10}$ 5	$\boxed{2}$ 10		
	$\boxed{7}$ 5	$\boxed{9}$ 15	$\boxed{20}$ 5
			$\boxed{18}$ 10

The starting basic soln is given as

$x_{11}=5,\ x_{12}=10,\ x_{22}=5,\ x_{23}=15,\ x_{24}=5,\ x_{34}=10$

The associated cost of the schedule is,

$$z = (5\times10)+(10\times2)+(5\times7)+(15\times9)+(5\times20)+(10\times18)$$

$$= 520.$$

Least cost method:

This method takes into account the minimum unit cost can be summarized.

Step: 1

Determine the smallest cost in the cost matrix of the transportation table. Let it be c_{ij}. Allocate $x_{ij} = \min(a_i, b_j)$ in the cell (i, j).

Step: 2

If $x_{ij} = a_i$ cross off the i^{th} row of the transportation table and decrease b_j by a_i. Go to step 3.

If $x_{ij} = b_j$ cross off the j^{th} column of the transportation table and decrease a_i by b_j. Go to step 3.

If $x_{ij} = a_i = b_j$ cross off either the i^{th} row or j^{th} column.

Step 3:

Repeat steps 1 and 2 for the resulting reduced transportation table until all the rim requirements are satisfied. Whenever the minimum cost is not unique, make an arbitrary choice among the minima.

Ex:

	1	2	3	4	Supply
1	[10] X	[2] 15	[20] X	[11] X	~~15~~
2	[12] X	[7] X	[9] 15	[20] 10	10 ~~25~~
3	[4] 5	[14] X	[16] X	[18] 5	~~10~~ 5
Demand	5	15	15	15 10	50

$$Z = 15\times 2 + 15\times 9 + 10\times 20 + 5\times 4 + 5\times 18$$

$$= 30 + 135 + 200 + 20 + 90$$

$$= 475$$

Vogel Approximation method (VAM)

VAM is an improved version of the least cost method that generally produces better starting solution.

Step : 1

For each row (column) determine a penalty measure by subtracting the smallest unit cost element in row (column) from the next smallest unit cost element in the same row (column)

Step : 2

Identify the row or column with the largest penalty. Break this arbitrary. Allocate as much as possible to the variable with the least unit cost in the selected row or column. Adjust the supply and demand and cross out the satisfied row (or) column. If a row and a column are satisfied simultaneously only one of the two is crossed out and the remaining row (column) is assigned zero supply (demand)

Step : 3

a) If exactly one row (column) with 0 supply or demand remains uncrossed out, stop.

b) If one row (column) with +ve supply (demand) remains uncrossed out determine the basic variables in the row (column) by the least-cost method

c) If all the uncrossed out rows and column have (remaining) zero supply and demand determine the zero basic variables by the least cost method. stop.

d) otherwise, go to step 1.

10	2 15	20 x	11 x	15	8	9
12 x	7	9 15	20 10	25	2	2 11 20
4 5	14 x	16 x	18 5	10	10	2 2 18
5	15	15	15	50		
6	5	7	7			
–	5	7	7			
–	–	7	2			
–	–	–	2			

$Z = 15 \times 2 + 15 \times 9 \times 10 \times 20 + 4 \times 5 + 18 \times 5$

$= 30 + 135 + 200 + 20 + 90$

$= 475$

a)

0	2	1	6
2	1	5	7
2	4	3	7
5	5	10	

Soln:

(i) North west corner method

The basic starting soln is $x_{11}=5$, $x_{12}=1$, $x_{22}=4$, $x_{23}=3$, $x_{33}=7$

0 5	2 1	1 x	6 1
2 x	1 4	5 3	7 3
2 x	4 x	3 7	7
5	5 4	10	20

$Z = 0 \times 5 + 1 \times 2 + 4 \times 1 + 5 \times 3 + 7 \times 3$

$= 0 + 2 + 4 + 15 + 21$

$= 42$

ii) least - cost method

0 5	2 x	1 1	6 x
2 x	1 5	5 2	7 2
2 x	4 x	3 7	7
5	5	10 9	20

$Z = 0 \times 5 + 1 \times 1 + 1 \times 5 + 5 \times 2 + 7 \times 3$

$= 0 + 1 + 5 + 10 + 21$

$= 37$

(iii) Vogel - approximation method.

[0] 5	[2] X	[1] 1	6, 1	1	1	1
[2] X	[1] 5	[5] 2	7, 2	1	4	5
[2] X	[4] X	[3] 7	7	1	1	3
5	5	~~10~~ 8				
2	1	2				
—	1	2				
—	—	2				

$Z = 0\times5 + 1\times1 + 5\times1 + 5\times2 + 7\times3$

$= 0 + 1 + 5 + 10 + 21$

$= 37$

b)

1	2	6	7
0	4	2	12
3	1	5	11
10	10	10	

Soln:

(i) North west corner method.

[1] 7	[2] X	[6] X	~~7~~
[0] 3	[4] 9	[2] X	~~12~~ 9
[3] X	[1] 1	[5] 10	~~11~~ 10
~~10~~ 3	~~10~~ 1	~~10~~	30

$Z = 1\times7 + 3\times0 + 9\times4 + 1\times1 + 5\times10$

$= 7 + 0 + 36 + 1 + 50$

$= 94$

(ii) Least cost method:

[1] X	[2] X	[6] 7	7
[0] 10	[4] X	[2] 2	~~12~~ 2
[3] X	[1] 10	[5] 1	11, 1
~~10~~	~~10~~	~~10~~ 8	

$Z = 6\times7 + 0\times10 + 2\times2 + 10\times1 + 5\times1$

$= 42 + 0 + 4 + 10 + 5$

$= 61$

iii) VAM method

$\boxed{1}$ 7	$\boxed{2}$ ×	$\boxed{6}$ ×	7	1	1	1	1
$\boxed{0}$ 2	$\boxed{4}$ ×	$\boxed{2}$ 10	$\cancel{12}$ 2	2	4	–	–
$\boxed{3}$ 1	$\boxed{1}$ 10	$\boxed{5}$ ×	$\cancel{11}$ 1	2	2	2	3
$\cancel{10}$ $\cancel{8}$ 7	10	10					
1	1	3					
1	1	–					
2	1	–					
2	–	–					

$Z = 1\times7 + 2\times0 + 10\times2 + 1\times3 + 10\times1$

$= 7 + 0 + 20 + 3 + 10$

$= 40$

c)

5	1	8	12
2	4	0	14
3	6	7	4
9	10	11	30

Soln: (i) North - west corner method.

$\boxed{5}$ 9	$\boxed{1}$ 3	$\boxed{8}$ ×	$\cancel{12}$ 3
$\boxed{2}$ ×	$\boxed{4}$ 7	$\boxed{0}$ 7	$\cancel{14}$ 7
$\boxed{3}$ ×	$\boxed{6}$ ×	$\boxed{7}$ 4	4
$\cancel{9}$	$\cancel{10}$ 7	11	30

$Z = 9\times5 + 3\times1 + 7\times4 + 7\times0 + 4\times7$

$= 45 + 3 + 28 + 0 + 28$

$= 104$

(ii) Least - cost method.

$\boxed{5}$ 2	$\boxed{1}$ 10	$\boxed{8}$ ×	$\cancel{12}$ 2
$\boxed{2}$ 3	$\boxed{4}$ ×	$\boxed{0}$ 11	$\cancel{14}$ 3
$\boxed{3}$ 4	$\boxed{6}$ ×	$\boxed{7}$ ×	4
$\cancel{9}$ 6	$\cancel{10}$	$\cancel{11}$	30

$Z = 5\times2 + 10\times1 + 3\times2 + 11\times0 + 4\times3$

$= 10 + 10 + 6 + 0 + 12$

$= 38.$

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(iii) VAM method :

$\boxed{5}$ 2	$\boxed{1}$ 10	$\boxed{8}$ X	12 (2)	4	4	5	-
$\boxed{2}$ 3	$\boxed{4}$ X	$\boxed{0}$ 11	14 (3)	2	2	2	2
$\boxed{3}$ 4	$\boxed{6}$ X	$\boxed{7}$ X	4	3	3	3	3
9 (7, 3)	10	11	30				
1	3	7					
1	3	-					
1	-	-					
1	-	-					

$Z = 2 \times 5 + 10 \times 1 + 3 \times 2 + 11 \times 0 + 3 \times 4$

$= 10 + 10 + 6 + 0 + 12$

$= 38.$

Transportation Algorithm (MODI method)

Various steps involved in solving any transportation problem may be summarized in the following iterative procedure.

Step : 1

Find the initial basic feasible soln by using any of the three methods discussed above

Step : 2

Check the no. of occupied cells. If there are less than $m+n-1$, there exists degeneracy and we introduce a very small positive assignment of $\varepsilon (\to 0)$ in suitable independent positions, so that the no. of occupied cells is exactly equal to $m+n-1$.

Step : 3

For each occupied cell in the current soln, solve the system of eqns $u_i + v_j = c_{ij}$, starting initially with some $u_i = 0$ (or) $v_j = 0$ and entering successively the values of u_i and v_j in the transportation table margins.

Step: 4

compute the net evaluations $z_{ij} = u_i + v_j - c_{ij}$ for all unoccupied basic cells and enter them in the lower left corners of the corresponding cells.

Step: 5

Examine the sign of each $z_{ij} - c_{ij}$

If all $z_{ij} - c_{ij} \leq 0$, Then The current basic feasible soln is an optimum one. If atleast one $z_{ij} - c_{ij} > 0$ select the unoccupied cell, having the largest +ve net evaluation enter the basis.

Step: 6

Let the unoccupied cell (r, s) enter the basis. Allocate an unknown quantity say θ to cell (r, s). Identify a loop that starts and end at the cell (r, s) and connects some of the basic cells. Add & subtract interchangeably, θ to and from the transition cells. If the loop in such a way that the rime requirements remain satisfied.

Step: 7

Assign a maximum value to θ in such a way that the value of one basic variable becomes zero and the other basic variables remain non-negatives. The basic cell whose allocation has been reduced to zero leaves the basis.

Step: 8

Return to step 3 and repeat the process until an optimum basic feasible soln has been obtained

Prob **Method of multiplier** (MODI method (or) u-v method).

Using north-west corner method, the initial basic feasible soln for the transportation problem.

	1	2	3	4	Supply
1	5 [10]	10 [2]	X [20]	X [11]	~~15~~ 10
2	X [12]	5 [7]	15 [9]	5 [20]	~~25~~ ~~20~~ 5
3	X [4]	X [14]	X [16]	10 [18]	10
Demand	~~5~~	~~15~~ 5	~~15~~	15	50

The total cost, $Z = 5 \times 10 + 10 \times 2 + 5 \times 7 + 15 \times 9 + 20 \times 5 + 18 \times 10$

$= 50 + 20 + 35 + 135 + 100 + 180$

$= 520.$

$m+n-1$ elements

To find an optimal soln:

We now compute $u_i + v_j = c_{ij}$ for the occupied cells.

Basic variable	(u, v) eqn	put $u_1 = 0$ soln
x_{11}	$u_1 + v_1 = 10$	$v_1 = 10$
x_{12}	$u_1 + v_2 = 2$	$v_2 = 2$
x_{22}	$u_2 + v_2 = 7$	$u_2 = 5$
x_{23}	$u_2 + v_3 = 9$	$v_3 = 4$
x_{24}	$u_2 + v_4 = 20$	$v_4 = 15$
x_{34}	$u_3 + v_4 = 18$	$u_3 = 3.$

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The net evaluation for each unoccupied cells.

non basic variable	$u_i + v_j - c_{ij}$
x_{13}	$u_1 + v_3 - c_{13} = 0 + 4 - 20 = -16$
x_{14}	$u_1 + v_4 - c_{14} = 0 + 15 - 11 = 4$
x_{21}	$u_2 + v_1 - c_{21} = 5 + 10 - 12 = 3$
x_{31}	$u_3 + v_1 - c_{31} = 3 + 10 - 4 = 9$
x_{32}	$u_3 + v_2 - c_{32} = 3 + 2 - 14 = -9$
x_{33}	$u_3 + v_3 - c_{33} = 3 + 4 - 16 = -9$

+ve value பார்க்கும் அதில் greatest value பார்க்க வேண்டும்,

According to optimality criterian, since some $z_{ij} - c_{ij} > 0$ current basic feasible soln is not optimum.

ஒரு +ve இருந்தாலும்

	$v_1 = 10$	$v_2 = 2$	$v_3 = 4$	$v_4 = 15$	Supply
$u_1 = 0$	[10] $5-\theta$	[2] $10+\theta$	[20]	[11]	15
$u_2 = 5$	[12]	[7] $5-\theta$	[9] 15	[20] $5+\theta$	25
$u_3 = 3$	[4] $+\theta$	[14]	[16]	[18] $10-\theta$	10
Demand	5	15	15	15	50

x_{31} enters the basic variable x_{11} leaves the basic variable.

$\theta = 5$

−θ போட்டதில் எது சிறிய value பார்க்க வேண்டும். அது θ

[10] 0	[2] 15	[20]	[11]
[12]	[7] 0	[9] 15	[20] 10
[4] 5	[14]	[16]	[18] 5

$Z = 15 \times 2 + 0 \times 7 + 15 \times 9 + 10 \times 20 + 5 \times 4 + 5 \times 18$

$= 30 + 135 + 200 + 20 + 90$

$= 475$

we now compute $u_i + v_j = c_{ij}$ for the occupied cells. put $u_1 = 0$.

$x_{12} = u_1 + v_2 = 2 \Rightarrow v_2 = 2$

$x_{22} = u_2 + v_2 = 7 \Rightarrow u_2 = 5$

$x_{23} = u_2 + v_3 = 9 \Rightarrow v_3 = 4$

$x_{24} = u_2 + v_4 = 20 \Rightarrow v_4 = 15$

$x_{31} = u_3 + v_1 = 4 \Rightarrow v_1 = 1$

$x_{34} = u_3 + v_4 = 18 \Rightarrow u_3 = 3$

The net evaluation for each unoccupied cells.

$x_{11} = u_1 + v_1 - c_{11} = 0 + 1 - 10 = -9$

$x_{13} = u_1 + v_3 - c_{13} = 0 + 4 - 20 = -16$

$x_{14} = u_1 + v_4 - c_{14} = 0 + 15 - 11 = 4 > 0$

$x_{21} = u_2 + v_1 - c_{21} = 5 + 1 - 12 = -6$

$x_{32} = u_3 + v_2 - c_{32} = 3 + 2 - 14 = -9$

$x_{33} = u_3 + v_3 - c_{33} = 3 + 4 - 16 = -9$

According to optimality critrian, since one $z_{ij} - c_{ij} > 0$, current basic feasible soln is not optimum.

10	2 $15 - \theta$	20	11 $+\theta$
12	7 $0 + \theta$	9 15	20 $10 - \theta$
4 5	14	16	18 5

x_{14} enters the basic variable x_{24} leave basic variable.

$\theta = 10$.

$\boxed{10}$	$\boxed{2}$ 5	$\boxed{20}$	$\boxed{11}$ 10
$\boxed{12}$	$\boxed{7}$ 10	$\boxed{9}$ 15	$\boxed{20}$
$\boxed{4}$ 5	$\boxed{14}$	$\boxed{16}$	$\boxed{18}$ 5

$Z = 5 \times 2 + 10 \times 11 + 10 \times 7 + 15 \times 9 + 5 \times 4 + 5 \times 18$

$= 10 + 110 + 70 + 135 + 20 + 90$

$= 435$

we now compute $u_i + v_j = c_{ij}$ for the occupied cells. put $u_1 = 0$.

$x_{12} = u_1 + v_2 = 2 \Rightarrow v_2 = 2$

$x_{22} = u_2 + v_2 = 7 \Rightarrow u_2 = 5$

$x_{23} = u_2 + v_3 = 9 \Rightarrow v_3 = 4$

$x_{31} = u_3 + v_1 = 4 \Rightarrow v_1 = -3$

$x_{34} = u_3 + v_4 = 18 \Rightarrow u_3 = 7$

$x_{14} = u_1 + v_4 = 11 \Rightarrow v_4 = 11$

The net evaluation for each unoccupied cells.

$x_{11} = u_1 + v_1 - c_{11} = 0 - 3 - 10 = -13$

$x_{13} = u_1 + v_3 - c_{13} = 0 + 4 - 20 = -16$

$x_{21} = u_2 + v_1 - c_{21} = 5 - 3 - 12 = -10$

$x_{24} = u_2 + v_4 - c_{24} = 5 + 11 - 20 = -4$

$x_{32} = u_3 + v_2 - c_{32} = 7 + 2 - 14 = -5$

$x_{33} = u_3 + v_3 - c_{33} = 7 + 4 - 16 = -5$

The new $u_i + v_j - c_{ij}$ are non-negative for all non-basic x_{ij}.

Thus the solution in new table is optimal

$\therefore$ The total optimal cost = 435.

Degeneracy in transportation problem

A basic feasible soln for the general transportation problem must consist of $(m+n-1)$ occupied cells. The basic solution will be called $\boxed{\text{degenerate}}$, when the no. of occupied cells is less than the required number $m+n-1$. Degeneracy can occur is initial soln or it may arise in some subsequent iteration. we now discuss a Procedure to deal with the problem of degeneracy.

Case :(i)

Degeneracy at the initial solution.

To resolve degeneracy at the initial soln a very small quantity $\varepsilon (>0)$ is allocated in an unoccuppied cell so as to get $m+n-1$ no. of unoccupied cells. In a minimization transportation cells that have lowest transportation cost. In some cases ε must be added in one of these unoccupied cells which make possible the determination of u_i and v_j uniquely.

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The quantity ε is considered to be so small that if is transferred to an occupied cell it does not change the quantity of allocation

ie) $x_{ij} + \varepsilon = x_{ij} - \varepsilon = x_{ij}$ but $\varepsilon - \varepsilon = 0$.

Also, ε does not change the quantity of allocation. The total transportation cost of the allocation.

Hence the quantity ε is used to evaluate unoccupied cells and once the purpose in over ε must be remove from the scene.

Case (ii)

Degeneracy at subsequent iterations

To resolve degeneracy which occur during optimality test the quantity ε may be allocated to one or more cells which have become unoccupied recently to have $m+n-1$ no. of occupied cells in the new soln. It may be removed once the purpose is over.

Unbalance transportation problem

In a transportation problem, if the demand exceeds the's supply or the supply exceeds the demand, then we will get a unbalance transportation model.

In this case $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$

Case (i)

If the demand exceeds the supply, then dummy sources with capacity = total demand - total supply is to be created and the cost of transportation from the dummy sources to all the destination is zero. This will convert the model into a balanced transported

Case (ii)

If the supply exceeds the demand, then the dummy destinations with the capacity = total supply − total demand is to be created and the cost of transportation from all the source to This dummy destination is zero.

The structure of the problem

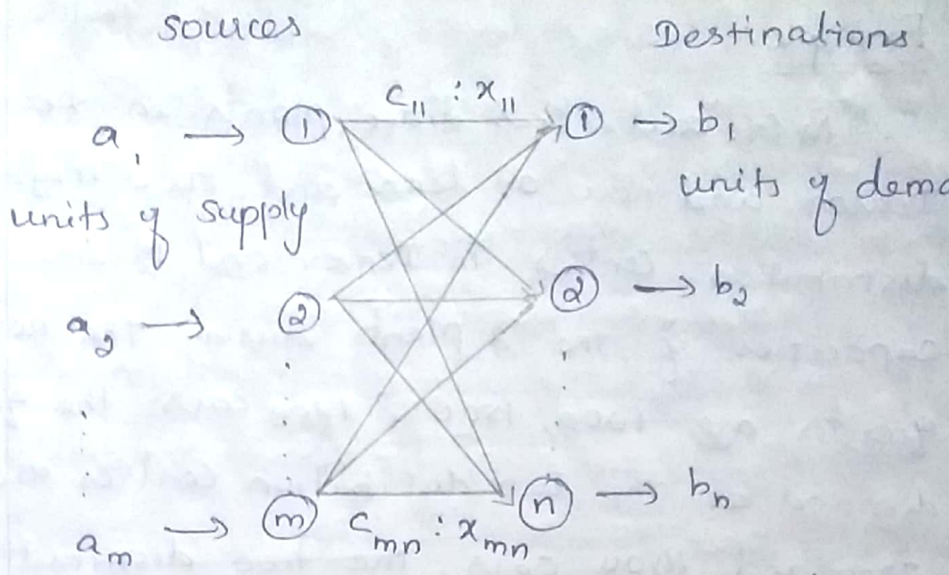
The general problem is represented by the network in the following figure. There are m sources and n destinations, each represented by a node. The arcs linking, the sources and destinations represent the routes between the sources and the destination.

Arc (i, j) joining source i to the destination j carries two places of information

(i) The transportation cost per unit C_{ij}.

(ii) The amount shipped x_{ij}.

The amount of supply at the source 'i' is a_i and the amount of demand at the destination 'j' is, b_j. The objective of the model is to determine the unknown x_{ij} that will minimize the total transportation cost while satisfying all the supply and demand restrictions.



The above information can be represented in a tabular form as follows.

Origins	Destination 1	2	3	....	n	Units of Supply
1	$c_{11} : x_{11}$	$c_{12} : x_{12}$	$c_{13} : x_{13}$	....	$c_{1n} : x_{1n}$	a_1
2	$c_{21} : x_{21}$	$c_{22} : x_{22}$	$c_{23} : x_{23}$	....	$c_{2n} : x_{2n}$	a_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
m	$c_{m1} : x_{m1}$	$c_{m2} : x_{m2}$	$c_{m3} : x_{m3}$	....	$c_{mn} : x_{mn}$	a_m
Units of demand	b_1	b_2	b_3	....	b_n	

The transportation model can be represented as minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$ subject to

$\sum_{j=1}^{n} x_{ij} = a_i$, $i = 1, 2, \ldots, m$; $\sum_{i=1}^{m} x_{ij} = b_j$, $j = 1, 2, \ldots, n$.

Balance transportation problem

In a transportation problem if the sum of supplier at the origin = the sum of the demand at the destinations, then the transportation problem is called a balanced transportation problem.

ie) $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

Example:

M.G. auto has three plants in Los angles Detroit and new or leans and two major distribution centres in Denver and Miami. The capacities of the 3 plants during the next quarter are 1000, 1500 & 1200 cars. The quantity demand at the two distribution centres are 2300 and 1400 cars. ~~The two distribution centres are 2300 ano~~ The mileage chart between the plants and the distribution in table 1.

	Denver	Miami
Los Angeles	1000	2690
Detroit	1250	1350
New orleans	1275	850

÷ by 12.5

The trucking company in charge of transporting the cars charges 8 cents per mile per car. Ther transportation cost per car on the different routes rounded to the closset dollar, are calculated as given in Table 2.

1 mile = 8 cent

1000 mileage = 8000 cents = $80

	Denver (1)	Miami (2)
Los Angels (1)	$ 80	$ 215
Detroit (2)	$ 100	$ 108
New orleans (3)	$ 102	$ 68

1 டாலர் = 100 செண்ட்

Soln:

Transportation table:

Destination centre

Plant	Denver	Mian	
L.A	$ 80	$ 215	1000
D	$ 100	$ 108	1500
N.O	$ 102	$ 68	1200
	2300	1400	3700

Let x_{ij} represents the no. of units shipped from Plant i to destination centre j.

Let z be the total cost

Then the L.P. model of the Problem is

$$\text{minimize } z = \overset{c_{11}}{80}x_{11} + \overset{c_{12}}{215}x_{12} + \overset{c_{21}}{100}x_{21} + \overset{c_{22}}{108}x_{22} + \overset{c_{31}}{102}x_{31} + \overset{c_{32}}{68}x_{32}$$

Subject to
$$x_{11} + x_{12} = 1000$$
$$x_{21} + x_{22} = 1500$$
$$x_{31} + x_{32} = 1200$$
$$x_{11} + x_{21} + x_{31} = 2300$$
$$x_{21} + x_{22} + x_{32} = 1400$$

Note:

These constraints are all eqns because the total supply from the 3 sources (1000 + 1500 + 1200 = 3700 cars) equals to the total demand at the two destinations (2300 + 1400 = 3700 cars).

Example:

(i) In MG model, suppose that the Detroit plant capacity is 1300 cars. This means that the total supply (3500 cars) is less than the total demand (3700 cars) a situation that dictates that part of the demand at denver may not be filled.

soln:

Total supply = 3500 cars

Total Demand = 3700 cars

Now, the Demand exceeds ~~with capacity~~ the supply.

∴ we create dummy source with capacity

= total demand − total supply

= 3700 − 3500

= 200 cars

In this case the unit transportation cost is 0 from the dummy plant to the two destinations because the plant does not exist. The following ~~tabble~~ table give the balanced model together with its optimum soln. The soln shows that the dummy plant ships 200 cars to Miami

(North west method)

	Denver	Miami	Supply
L.A	80 / 1000	215 / X	1000
Detroit	100 / 1300	108 / X	1300
N.O	102 / X	68 / 1200	1200
Dummy Plant	0 / X	0 / 200	200
Demand	~~2300~~ 1300	~~1400~~ 200	3700

$Z = 1000 \times 80 + 1300 \times 100 + 1200 \times 68 + 200 \times 0$

$= 80000 + 130000 + 81600 + 0$

$= 2,91,600$

(ii) In MG model, assume that the demand at Denvar is 1900 cars only. Then construct the T.M.

<u>soln</u>:

Total supply = 3700 cars

Total Demand = 3300 cars

supply exceeds the demand

∴ we create a dummy destination with capacity

= Total supply - Total demand

= 3700 - 3300

= 400 cars

In This, the unit transportation costs to The dummy distribution centre is 0

The following table gives the new model and its optimal soln. The soln shows that the Detroit surplus of 400 cars

	Denver	Miami	dummy destination	supply
L.A	80 1000	215 ×	0 ×	~~1000~~
Detroit	100 900	108 600	0 ×	~~1500~~ 600
N.O	102 ×	68 800	0 400	~~1200~~ 400
Demand	~~1900~~ 900	~~1400~~ 800	400	3700

$Z = 1000 \times 80 + 900 \times 100 + 600 \times 108 + 800 \times 68 + 0 \times 400$

$= 80000 + 90000 + 64800 + 54400 + 0$

$= 2,89,200$

Prob. 2). In each of the following cases, determine whether a dummy source or a dummy destination must be added to balance the model

(a) supply $a_1 = 10$, $a_2 = 5$, $a_3 = 4$, $a_4 = 6$.

Demand $b_1 = 10$, $b_2 = 5$, $b_3 = 7$, $b_4 = 9$

Soln:
Here the total supply $= 10 + 5 + 4 + 6 = 25$

the total Demand $= 10 + 5 + 7 + 9 = 31$

$\therefore$ we create a dummy source with capacity

$=$ total demand $-$ total supply

$= 31 - 25$

$= 6$

	a_1	a_2	a_3	a_4	dummy source	
b_1					0	10
b_2					0	5
b_3					0	7
b_4					0	9
	10	5	4	6	6	31

(b) Supply $a_1 = 30$, $a_2 = 44$

Demand $b_1 = 25$, $b_2 = 30$, $b_3 = 10$

soln:

Total supply $= 74$

Total Demand $= 65$

$\therefore$ we create a dummy destination with

Capacity = total supply − total demand

$= 74 - 65$

$= 9$

	a_1	a_2	supply
b_1			25
b_2			30
b_3			10
dummy destination	0 ~~300~~	0 ~~400~~	9
Demand	30	44	74

Ex. Prob. 1. (UV method) In the transportation models, use the northwest Corner method to find the stating soln, the determine the optimum soln.

a)

0 $\boxed{5}$	2 $\boxed{1}$	1 x	~~6~~ 1	
2 x	1 $\boxed{4}$	5 $\boxed{5}$	~~9~~ 5	
2 x	4 x	3 $\boxed{5}$	5	
~~5~~	~~5~~ 4	10	20	

$Z = 0\times5 + 2\times1 + 1\times4 + 5\times5 + 3\times5$

$= 0 + 2 + 4 + 25 + 15$

$= 46$

To find optimum soln, we now compute

$u_i + v_j = c_{ij}$ for occupied cells

put $u_1 = 0$, soln

Basic variable	(u, v) eqn	
		$v_1 = 0$
x_{11}	$u_1 + v_1 = 0$	$v_2 = 2$
x_{12}	$u_1 + v_2 = 2$	$u_2 = -1$
x_{22}	$u_2 + v_2 = 1$	$v_3 = 6$
x_{23}	$u_2 + v_3 = 5$	$u_3 = -3$
x_{33}	$u_3 + v_3 = 3$	

The net evaluation for each non-basic variables,

$u_i + v_j - c_{ij}$

non basic variables	$u_i + v_j - c_{ij}$
x_{13}	$u_1 + v_3 - 1 = 0 + 6 - 1 = 5$
x_{21}	$u_2 + v_1 - 2 = -1 + 0 - 2 = -3$
x_{31}	$u_3 + v_1 - 2 = -3 + 0 - 2 = -5$
x_{32}	$u_3 + v_2 - 4 = -3 + 2 - 4 = -5$

According to optimality criterion, since some $z_{ij} - c_{ij} > 0$ current feasible soln is not optimum

$\theta = 1$ (the smallest values in occupied values)

0 5	2 $1 - \theta$	1 $+\theta$
2	1 $4 + \theta$	5 $5 - \theta$
2	4	3 5

$m + n - 1$

$3 + 3 - 1$

5

0 5	2	1 1
2	1 5	5 4
2	4	3 5

$Z = 5\times 0 + 1\times 1 + 1\times 5 + 4\times 5 + 3\times 5$

$= 0 + 1 + 5 + 20 + 15$

$= 41$

To find optimum soln, we now compute $u_i + v_j = c_{ij}$ for occupied cells.

Basic variable	(u, v) eqn	put $u_1 = 0$ soln
x_{11}	$u_1 + v_1 = 0$	$v_1 = 0$
x_{13}	$u_1 + v_3 = 1$	$v_3 = 1$
x_{22}	$u_2 + v_2 = 1$	$v_2 = -3$
x_{23}	$u_2 + v_3 = 5$	$u_2 = 4$
x_{33}	$u_3 + v_3 = 3$	$u_3 = 2$

The net evaluation for each unoccupied cells.

non-basic variable	$u_i + v_j - c_{ij}$
x_{12}	$u_1 + v_2 - 2 \Rightarrow 0 + (-3) - 2 = -5$
x_{21}	$u_2 + v_1 - 2 \Rightarrow 4 + 0 - 2 = 2$
x_{31}	$u_3 + v_1 - 2 \Rightarrow 2 + 0 - 2 = 0$
x_{32}	$u_3 + v_2 - 4 \Rightarrow 2 - 3 - 4 = -5$

According to optimality criterion, since one $z_{ij} - c_{ij} > 0$, the current feasible soln is not optimum

$\theta = 4$

[0] $5-\theta$	[2]	[1] $1+\theta$
[2] $+\theta$	[1] $4-\theta$	[5] $5-\theta$
[2]	[4]	[3] 5

[0] 0	[2]	[1] 6
[2] 5	[1] 4	[5] 0
[2]	[4]	[3] 5

$$Z = 0\times\cancel{4} + 5\times 2 + 4\times 2 + 5\times 5 + 5\times 3$$
$$= 0 + 10 + 8 + 25 + 15$$
$$= 58$$

$\boxed{Z = 6\times 1 + 2\times 5 + 1\times 4 + 5\times 3}$

$\boxed{Z = 35}$

28
15
35

To find optimum soln, we now compute $u_i + v_j = c_{ij}$ for occupied cells.

Basic variable	(u, v) eqn	put $u_1 = 0$
x_{11}	$u_1 + v_1 = 0$	$v_1 = 0$
x_{12}	$u_1 + v_2 = 2$	$v_2 = 2$
x_{21}	$u_2 + v_1 = 2$	$u_2 = 2$
x_{23}	$u_2 + v_3 = 5$	$v_3 = 3$
x_{33}	$u_3 + v_3 = 3$	$u_3 = 0$

The net evaluation for each unoccupied cells.

non-basic variable	$u_i + v_j - c_{ij}$
x_{13}	$u_1 + v_3 - 1 \Rightarrow 0 + 3 - 1 = 2$
x_{22}	$u_2 + v_2 - 1 \Rightarrow$
x_{31}	$u_3 + v_1 - 2$
x_{32}	$u_3 + v_2 - 4$

Basic variable	(u, v) eqn	put $u_1 = 0$
~~x_{11}~~	~~$u_1 + v_1 = 0$~~	
x_{13}	$u_1 + v_3 = 1$	$v_3 = 1$
x_{21}	$u_2 + v_1 = 2$	$v_1 = -2$
x_{22}	$u_2 + v_2 = 1$	$v_2 = -3$
x_{23}	$u_2 + v_3 = 5$	$u_2 = 4$
x_{33}	$u_3 + v_3 = 3$	$u_3 = 2$

non basic variable	$u_i + v_j - c_{ij}$
x_{11}	$u_1 + v_1 - 0 \Rightarrow 0 - 2 - 0 = -2$
x_{12}	$u_1 + v_2 - 2 \Rightarrow 0 - 3 - 2 = -5$
x_{31}	$u_3 + v_1 - 2 \Rightarrow 2 - 2 - 2 = -2$
x_{32}	$u_3 + v_2 - 4 \Rightarrow 2 - 3 - 4 = -5$

Now, $z_{ij} - c_{ij} \leq 0$ for all non basic variable

Thus the soln is optimum.

The total optimal cost $z = 35$

(b)

0 [7]	4 [1]	2 x	~~8~~ 1
2 x	3 [5]	4	5
1 x	2 [0]	0 [6]	6
7	6 (5)	6	19

$m+n-1$

$3+3-1$

5

soln

$Z = 0\times7 + 4\times1 + 3\times5 + 0\times2 + 0\times6$

$= 0 + 4 + 15 + 0 + 0$

$= 19.$

To find optimum soln, we now compute $u_i + v_j = c_{ij}$ for occupied cells.

Put $u_1 = 0$

basic variables.	(u, v) eqn	
x_{11}	$u_1 + v_1 = 0$	$v_1 = 0$
x_{12}	$u_1 + v_2 = 4$	$v_2 = 4$
x_{22}	$u_2 + v_2 = 3$	$u_2 = -1$
x_{32}	$u_3 + v_2 = 2$	$u_3 = -2$
x_{33}	$u_3 + v_3 = 0$	$v_3 = 2$

The net evaluation for unoccupied cells.

non basic variable	$u_i + v_j - c_{ij}$	
x_{13}	$u_1 + v_3 - 2$	$= 0 + 2 - 2 = 0$
x_{21}	$u_2 + v_1 - 2$	$= -1 + 0 - 2 = -3$
x_{23}	$u_2 + v_3 - 4$	$= -1 + 2 - 4 = -3$
x_{31}	$u_3 + v_1 - 1$	$= -2 + 0 - 1 = -3$

Here all $z_{ij} - c_{ij} \leq 0$. The soln is optimum

$\therefore$ The total optimal cost $z = 19$

Prob 2. In the transportation problem, the total demand exceeds the total supply. Suppose that the penalty costs per unit of unsatisfied demand are \$5, \$3 & \$2 for destination 1, 2 and 3 respectively. Determine optimum soln.

\$5	\$1	\$7	10
\$6	\$4	\$6	80
\$3	\$2	\$5	15
75	20	50	115 / 145

soln:

Here $\sum_{i=1}^{3} a_i \neq \sum_{j=1}^{3} b_j$

Here the transportation problem is unbalanced.

So we introduce a dummy zero cost.

5 ×	1 10	7 ×	10	4	4	–
6 60	4 10	6 10	~~80~~ 90	2	2	2
3 15	2 ×	5 ×	15	1	1	–
0 ×	0 ×	0 40	40	0	–	–
~~75~~ 60	~~20~~ 10	~~50~~ 10	145			
3	1	5				
2	1	1				
3	2	1				
–	2	1				
6	4	6				

Total cost, $z = 10\times1 + 60\times6 + 10\times4 + 10\times6 + 15\times3 + 0\times40$

$= 10 + 360 + 40 + 60 + 45 + 0$

$= \$515$

To find optimum soln:

we compute $u_i + v_j = c_{ij}$ for occupied cells.

Basic variable	(u,v) eqn	put $u_1 = 0$
x_{12}	$u_1 + v_2 = 1$	$v_2 = 1$
x_{21}	$u_2 + v_1 = 6$	$v_1 = 3$
x_{22}	$u_2 + v_2 = 4$	$u_2 = 3$
x_{23}	$u_2 + v_3 = 6$	$v_3 = 3$
x_{31}	$u_3 + v_1 = 3$	$u_3 = 0$
x_{43}	$u_4 + v_3 = 0$	$u_4 = -3$

The net evaluation for unoccupied cells.

non basic variable	$u_i + v_j - c_{ij}$
x_{11}	$u_1 + v_1 - 5 \Rightarrow 0+3-5 = -2$
x_{13}	$u_1 + v_3 - 7 \Rightarrow 0+3-7 = -4$
x_{32}	$u_3 + v_2 - 2 \Rightarrow 0+1-2 = -1$
x_{33}	$u_3 + v_3 - 5 \Rightarrow 0+3-5 = -2$
x_{41}	$u_4 + v_1 - 0 \Rightarrow -3+3-0 = 0$
x_{42}	$u_4 + v_2 - 0 \Rightarrow -3+1-0 = -2$

Here all $Z_{ij} - C_{ij} \leq 0$ for all unoccupied cells.

$\therefore$ The soln is optimum.

Hence the total cost $z = 515$.

Prob. 5. In a 3x3 transportation problem. The amounts of supply at sources 1,2 and 3 are 15, 30, 85 units respectively. and the demands at destinations 1, 2 & 3 are 20, 30 & 80 units respectively. Assume that the starting north-west corner soln is optimal and the associated values of the multipliers are given as $u_1 = -2$, $u_2 = 3$, $u_3 = 5$, $v_1 = 2$, $v_2 = 5$, $v_3 = 10$

a) Find the associated optimal cost.

b) Determine the smallest value of c_{ij} associated with each non-basic variable that will maintain the optimality of the north west corner method.

(a)

	1	2	3	supply
1	15	x	x	15
2	5	25	x	30 25
3	x	5	80	85
	20 5	30 5	80	130

By the north-west corner method, the starting soln is $x_{11} = 15$, $x_{21} = 5$, $x_{22} = 25$, $x_{32} = 5$, $x_{33} = 80$.

Now, we shall find the cost c_{ij} for the above basic variables. For the basic variables, we have

$c_{ij} = u_i + v_j$.

$c_{11} = u_1 + v_1 = -2 + 2 = 0$

$c_{21} = u_2 + v_1 = 3 + 2 = 5$

$c_{22} = u_2 + v_2 = 3 + 5 = 8$

$c_{32} = u_3 + v_2 = 5 + 5 = 10$

$c_{33} = u_3 + v_3 = 5 + 10 = 15$

$\therefore$ The associated optimal cost,

$z = 0 \times 15 + 5 \times 5 + 8 \times 25 + 10 \times 5 + 15 \times 80$

$= 0 + 25 + 200 + 50 + 1200$

$= 1475$

(b) For the non-basic variable that will maintain the optimal, we have $u_i + v_j - c_{ij} \leq 0$

ie) $c_{ij} - u_i - v_j \geq 0$

$c_{12} - u_1 - v_2 \geq 0 \Rightarrow c_{12} + 2 - 5 \geq 0$

$c_{12} \geq 3$

$\therefore$ The smallest value of $c_{12} = 3$

$c_{13} - u_1 - v_3 \geq 0 \Rightarrow c_{13} + 2 - 10 \geq 0$

$c_{13} \geq 8$

The smallest value of $c_{13} = 8$

$c_{23} - u_2 - v_3 \geq 0 \Rightarrow c_{23} - 3 - 10 \geq 0$

$c_{23} \geq 13$

$\therefore$ The smallest value of $c_{23} \geq 13$.

$c_{31} - u_3 - v_1 \geq 0 \Rightarrow c_{31} - 5 - 2 \geq 0$

$c_{31} \geq 7$

The smallest value of $c_{31} = 7$

The Assignment Problem

2015 (5m)

Suppose there are n worker to be done by n job. Let c_{ij} be the cost in curred when worker i is Processed by job j. The problem is to determine the assignment of worker to jobs such that one worker is assigned to one job and over all cost is minimum. This problem is Known as assignment problem.

The general assignment model with n workers and n jobs is represented in the table.

jobs

		1	2	....	n
	1	c_{11}	c_{12}	....	c_{1n}
	2	c_{21}	c_{22}	....	c_{2n}
workers	:	.	.		.
	n	c_{1n}	c_{n2}	....	c_{nn}

The elements c_{ij} represents the cost of assigning worker i to job j $(i, j = 1, 2, .., n)$ There is no loss in generality in assuming that the no. of workers always equal to the no. of jobs we can always add fictitious workers (or fictitious jobs to effect This result.

The assignment model is actually a special case of the transportation model in which the workers represents the sources and the job represents the destination.

The supply amount at each source and the demand amount at each destination equals 1

The L.P model of the assignment Problem is minimize $z = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij}$

Subject to $\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n$

$\sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n$

where $x_{ij} = 0$ (or) $1 \quad \forall i$

The Hungarian method

Procedure for solving an assignment pbm using Hungarian method.

Step: 1 : (Perfect square)

Prepare the cost matrix. If it is not a square, add dummy rows (or) dummy columns with zero cost.

Step: 2

For the orginal cost matrix, identify each row is minimum and subtract it from all the entries of the row.

Step: 3

For the resulting matrix, identify each column is minimum and subtract it from all the entries of the column.

Step: 4

* Identify the optimal assignment as the one associated with the zero element of the matrix obtained in step: 3

* If there is no feasible soln go to next step.

Step 5

(i) Draw the minimum no of horizontal and vertical lines in the last reduced matrix that will cover all the zero entries.

(ii) select the smallest uncovered element and subtract it from every ~~element~~ uncovered element, then add it to every element at the intersection of two line.

(iii) If no feasible assignment can be found among the resulting zero entries repeat step. 5 otherwise go to step 4 to determine the optimal assignment.

Example:

	Mow	paint	wash
John	15	10	9
karan	9	15	10
Terri	10	12	8

Soln: The cost matrix is a square matrix of order 3.

Hence $n = 3$.

Row minimum subtraction:

	Mow	paint	wash
John	6	1	0
karan	0	6	1
Terri	2	4	0

Colum minimum subtraction and give assignment

	Mow	paint	wash
John	6	0	0
karan	0	5	1
Terri	2	3	0

∴ No of assignment = 3 = order of the matrix

∴ The assignment is optimal.

Thus the optimum assignment is made as

J - P , K → M , T → W

The minimum Processing cost, z = 10 + 9 + 8

= 27

Ex:

Chore

		1	2	3	4
Child	1	\$1	\$4	\$6	\$3
	2	\$9	\$7	\$10	\$9
	3	\$4	\$5	\$11	\$7
	4	\$8	\$7	\$8	\$5

Soln:

The cost matrix is a square matrix of order 4

Hence n = 4

Row minimum subtraction:

	1	2	3	4
1	0	3	5	2
2	2	0	3	2
3	0	1	7	3
4	3	2	3	0

Column minimum subtraction and give assignment:

	1	2	3	4
1	0	3	2	2
2	2 +1	0	0	2
3	0	$\boxed{1}$	4	3
4	3 +1	2	0	0

Draw a minimum no. of lines covering all the zeros.

No of assignment = 3 ≠ order of the matrix

	1	2	3	4
1	$\boxed{0}$	2	1	1
2	3	0	$\boxed{0}$	2
3	0	$\boxed{0}$	3	2
4	4	2	0	$\boxed{0}$

No of assignment = 4 = order of the matrix

∴ The assignment is optimum.

Thus the optimum assignment is mad as

1 – 1, 2 → 3, 3 → 2, 4 → 4

∴ The minimum Processing cost,

$z = 1 + 10 + 5 + 5$

$= 21$

Prob. 1. Solve the assignment model

\$3	\$8	\$2	\$10	\$3
\$8	\$7	\$2	\$9	\$7
\$6	\$4	\$2	\$7	\$5
\$8	\$4	\$2	\$3	\$5
\$9	\$10	\$6	\$9	\$10

Soln: The cost matrix is a square matrix of order 5. ∴ n=5

Row minimum subtraction:

1	6	0	8	1
6	5	0	7	5
4	2	0	5	3
6	2	0	1	3
3	4	0	3	4

Column minimum subtraction:

0	4	0	7	0
5	3	0	6	4
3	0	0	4	2
5	0	0	0	2
2	2	0	2	3

Draw a minimum no. of lines covering all the zeros.

No. of assignment = 4 ≠ order of the matrix.

	1	2	3	4	5
1	0	4	2	7	$\boxed{0}$
2	3	1	$\boxed{0}$	4	2
3	3	$\boxed{0}$	2	4	2
4	5	0	2	$\boxed{0}$	2
5	$\boxed{0}$	0	0	0	3

No. of assignment = 5 = order of the matrix

The assignment is optimum.

The assignment is made as

Thus the optimum assignment is

1 → 5, 2 → 3, 3 → 2, 4 → 4, 5 → 1

The minimum processing cost, Z = 3 + 2 + 4 + 3 + 9
= 21

2)

3	9	2	3	7
6	1	5	6	6
9	4	7	10	3
2	5	4	2	1
9	6	2	4	5

Soln: The cost matrix is a square matrix of order 5.

Hence n = 5.

Row minimum subtraction:

1	7	0	1	5
5	0	4	5	5
6	1	4	7	0
1	4	3	1	0
7	4	0	2	3

Column minimum subtraction:

$$\begin{bmatrix} 0 & 7 & 0 & 0 & 5 \\ 4 & \boxed{0} & 4 & 4 & 5 \\ 5 & 1 & 4 & 6 & 0 \\ 0 & 4 & 3 & 0 & 0 \\ 6 & 4 & \boxed{0} & 1 & 3 \end{bmatrix}$$

$$\begin{matrix} \boxed{0} & 7 & 1 & 0 & 6 \\ 4 & \boxed{0} & 5 & 4 & 6 \\ 4 & 0 & 4 & 5 & \boxed{0} \\ 0 & 4 & 4 & \boxed{0} & 1 \\ 5 & 3 & \boxed{0} & 0 & 3 \end{matrix}$$

No. of assignment = 5 = order of matrix.

The assignment is optimal.

Thus the optimum assignment is made as

$1 \to 1, \; 2 \to 2, \; 3 \to 5, \; 4 \to 4, \; 5 \to 3$

The minimum Processing cost $Z = 3 + 1 + 3 + 2 + 2$

$= 11$

Prob.

Destination

		1	2	3	capacity
Sources	1	2 $\boxed{10}$	2 ×	3 ×	~~10~~
	2	$4-\theta$ $\boxed{10}$	$1+\theta$ $\boxed{5}$	2 ×	~~15~~ 5
	3	$+\theta$ 1 ×	$3-\theta$ $\boxed{10}$	1 $\boxed{30}$	40
		~~20~~ 10	~~15~~ 10	30	65

$m + n - 1$

$3 + 3 - 1$

$= 5$

Here,

$\square$ - 2mm number

θ - + (or) -

Soln:

$\sum a_i = \sum b_j = 65.$

$\therefore$ The pbm is balanced transportation pbm

Now, we find initial basic feasible soln using north-west corner method.

$Z = 2 \times 10 + 10 \times 4 + 1 \times 5 + 3 \times 10 + 1 \times 30$

$= 20 + 40 + 5 + 30 + 30$

$= 125$

To find an optimal soln:

we now compute $u_i + v_j = c_{ij}$ for the occupied cells.

basic Variables	(u, v) eqn	put $u_1 = 0$
x_{11}	$u_1 + v_1 = 2$	$v_1 = 2$
x_{21}	$u_2 + v_1 = 4$	$u_2 = 2$
x_{22}	$u_2 + v_2 = 1$	$v_2 = -1$
x_{32}	$u_3 + v_2 = 3$	$u_3 = 4$
x_{33}	$u_3 + v_3 = 1$	$v_3 = -3$

The net evaluation for unoccupied cells

non basic variable	$u_i + v_j - c_{ij}$
x_{12}	$u_1 + v_2 - 2 = 0 - 1 - 2 = -3$
x_{13}	$u_1 + v_3 - 3 = 0 - 3 - 3 = -6$
x_{23}	$u_2 + v_3 - 2 = 2 - 3 - 2 = -3$
x_{31}	$u_3 + v_1 - 1 = 4 + 2 - 1 = 5$

According to optimality condition, since one $z_{ij} - c_{ij} > 0$, the current basic feasible soln is not optimum. $\boxed{\theta = 10}$

10, 2	2	3
10, 4	5, 4	2
1	70, 3	30

2, $10-\theta$	2, $+\theta$	3
4, $\not{\alpha}$	1, 15	2
1, $10+\theta$	3, $0-\theta$	1, 30

x_{21}, x_{32} இரண்டும் 0 ஆகும்
occupied cells 4
ie) $m+n-1 = 4 < 5$
$3+3-5$
unbalanced

x_{32} leaves the variable & x_{32} enters the variable

$Z = 10 \times 2 + 0 \times 3 + 1 \times 15 + 10 \times 1 + 1 \times 30$

$= 20 + 0 + 15 + 10 + 30$

$= 75$

now, compute $u_i + v_j$ for occupied cells.

basic variable	(u, v) eqn	Put $u_1 = 0$
x_{11}	$u_1 + v_1 = 2$	$v_1 = 2$
~~x_{21}~~	~~$u_2 + v_1 = 4$~~	~~$u_2 = 2$~~
x_{22}	$u_2 + v_2 = 1$	$u_2 = -3$
x_{31}	$u_3 + v_1 = 1$	$u_3 = -1$
x_{32}	$u_3 + v_2 = 3$	$v_2 = 4$
x_{33}	$u_3 + v_3 = 1$	$v_3 = 2$

The net evaluation for unoccupied cells

non basic variables	$u_i + v_j - c_{ij}$
x_{12}	$u_1 + v_2 - 2 \Rightarrow 0 + 4 - 2 = 2$
x_{13}	$u_1 + v_3 - 3 \Rightarrow 0 + 2 - 3 = -1$
x_{21}	$u_2 + v_1 - 4 \Rightarrow -3 + 2 - 4 = -5$
x_{23}	$u_2 + v_3 - 2 \Rightarrow -3 + 2 - 2 = -3$

Since one $z_{ij} - c_{ij} > 0$, the current soln is not optimum

$\theta = 0$

x_{12} enters & x_{32} leaves.

2 / 10	2 / 0	3
4	1 / 15	2
1 / 10	3	1 / 30

$z = 10 \times 2 + 0 + 15 + 10 + 30$

$= 20 + 15 + 10 + 30$

$= 75$

I/ basic variable — (u, v) eqn — put $u_1 = 0$

basic variable	(u, v) eqn	put $u_1 = 0$
x_{11}	$u_1 + v_1 = 2$	$v_1 = 2$
x_{12}	$u_1 + v_2 = 2$	$v_2 = 2$
x_{22}	$u_2 + v_2 = 1$	$u_2 = -1$
x_{31}	$u_3 + v_1 = 1$	$u_3 = -1$
x_{33}	$u_3 + v_3 = 1$	$v_3 = 2$

The net evaluation for unoccupied cells.

x_{13}	$u_1 + v_3 - 3 = 0 + 2 - 3 = -1$
x_{21}	$u_2 + v_1 - 4 = -1 + 2 - 4 = -3$
x_{23}	$u_2 + v_3 - 2 = -1 + 2 - 2 = -1$
x_{32}	$u_3 + v_2 - 3 = -1 + 2 - 3 = -2$

Since all $z_{ij} - c_{ij} \leq 0$, the current feasible soln is optimum.) I

$\therefore z = 75$!)

Ex. of Degeneracy transportation pbm.

$m + n - 1 = 4 < 5$

$\therefore$ unbalanced transportation

2 10	2 ξ_1	3
4	1 15	2
1 10	3	1 30

Two occepied cells (2, 1) & (3, 2) becomes empty and the cell (3, 1) is occepied.

Resulting in a degenerate soln

The no. of occepied cells $= 4 < m+n-1 = 5$

We now, compute $u_i + v_j = c_{ij}$ for The occepied cells

write (I)

$x_{11} = 10$, $x_{12} = \xi_1$, $x_{22} = 15$, $x_{31} = 10$, $x_{33} = 30$

$\therefore$ The total transportation cost,

$$Z = 10 \times 2 + 2 \times \xi_1 + 15 \times 1 + 10 \times 1 + 30 \times 1$$

$$= 20 + 2\xi_1 + 15 + 10 + 30$$

$$= 75 + 2\xi_1$$

Since $\xi_1 > 0$ is arbitrary.

$\therefore$ The total transportation cost $Z = 75$